Sections 10.2 & 10.3

Grade: «grade»

Subject: «subject»

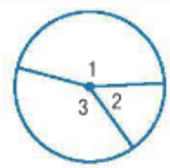
Date: «date»

A **central angle** has the center of the circle as its vertex, and its sides contain two radii of the circle.

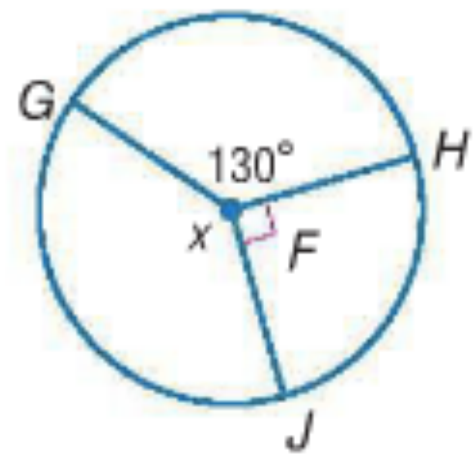
KeyConcept Sum of Central Angles

Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$



Find the value of x.

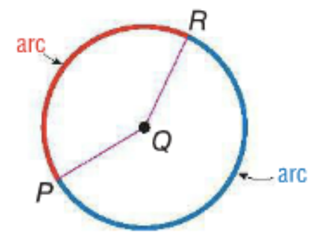


1 **Answer?**

$$x + 130 + 90 = 360$$

$$x = 140^\circ$$

An **arc** is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.

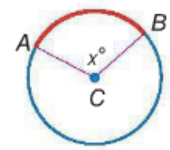
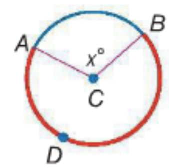
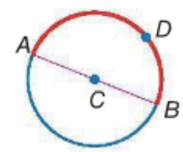


Arc and Central Angle Investigation

StudyTip

Naming Arcs Minor arcs are named by their endpoints. Major arcs and semicircles are named by their endpoints and another point on the arc that lies between these endpoints.

KeyConcept Arcs and Arc Measure

Arc	Measure	
A **minor arc** is the shortest arc connecting two endpoints on a circle.	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\overset{\frown}{AB} = m\angle ACB = x$	
A **major arc** is the longest arc connecting two endpoints on a circle.	The measure of a major arc is greater than 180, and equal to 360 minus the measure of the minor arc with the same endpoints. $m\overset{\frown}{ADB} = 360 - m\overset{\frown}{AB} = 360 - x$	
A **semicircle** is an arc with endpoints that lie on a diameter.	The measure of a semicircle is 180. $m\overset{\frown}{ADB} = 180$	

$\overline{GJ}$ is a diameter of $\odot K$. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

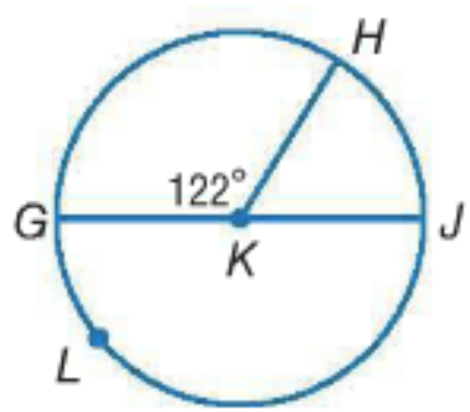
2 Answer?

A Major Arc - 238º

B Minor Arc - 122º

C Semi Circle - 180º

a. $m\widehat{GH}$

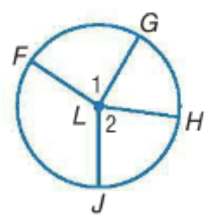


Congruent arcs are arcs in the same or congruent circles that have the same measure.

Theorem 10.1

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\widehat{FG} \cong \widehat{HJ}$.
If $\widehat{FG} \cong \widehat{HJ}$, then $\angle 1 \cong \angle 2$.



SPORTS Refer to the circle graph. Find each measure.

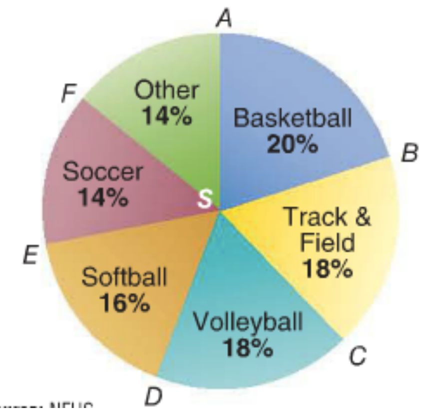
a. $m\widehat{CD}$

$.18(360) = 64.8°$

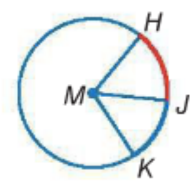
b. $m\widehat{BC}$

$64.8°$

Female Participation in Sports



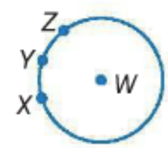
Adjacent arcs are arcs in a circle that have exactly one point in common. In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.

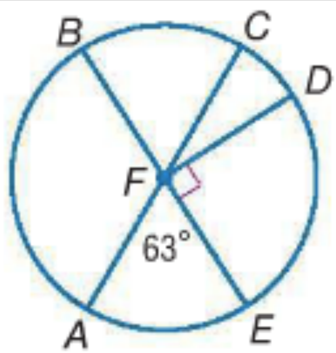


Postulate 10.1 Arc Addition Postulate

Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$





Find each measure in $\odot F$.

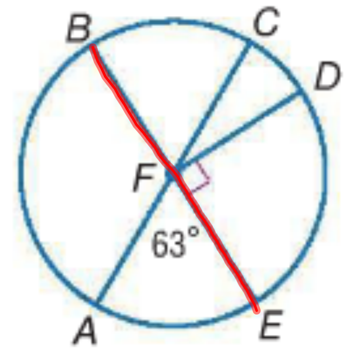
a. $m\widehat{AED}$

3 Answer?

$m\widehat{AE} + m\widehat{ED} = m\widehat{AD}$

$63 + 90 = m\widehat{AD} = 153^\circ$

b. $m\overset{\frown}{ADB}$



4 Answer?

$\overline{BE}$ is a diameter

$\overset{\frown}{BDE}$ is semicircle $= 180°$

$m\overset{\frown}{BDE} + m\overset{\frown}{AE} = m\overset{\frown}{ADB}$

$180 + 63 = m\overset{\frown}{ADB} = 243°$

2 Arc Length **Arc length** is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

Length of an Arc

ratio of lengths

$$\frac{\text{length arc}}{C = 2\pi r \ (\text{length of circle})} = \frac{\text{degree of arc (central angle)}}{360 \ (\text{degree of circle})}$$

ratio of degrees

Key Concept — Arc Length

$$\frac{A}{360} = \frac{\ell}{2\pi r}$$

degree measure of arc → A; degree measure of whole circle → 360; ℓ ← arc length; $2\pi r$ ← circumference

This can also be expressed as $\frac{A}{360} \cdot C = \ell$.

WatchOut!

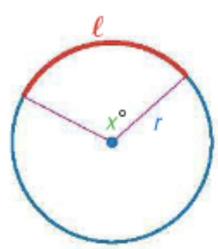
Arc Length The length of an arc is given in linear units, such as centimeters. The measure of an arc is given in degrees.

KeyConcept Arc Length

Words The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.

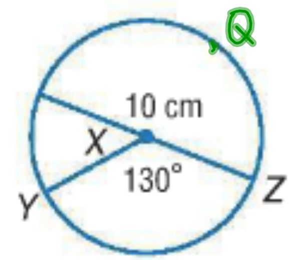
Proportion $\frac{\ell}{2\pi r} = \frac{x}{360}$ or

Equation $\ell = \frac{x}{360} \cdot 2\pi r$



Find the length of $\widehat{ZY}$. Round to the nearest hundredth.

b.



5 Answer (rounded to nearest hundreth)?

$$\text{lengths}\left\{\frac{\ell}{10\pi}\right. = \left.\frac{130}{360}\right\}\text{degrees}$$

$$360\ell = 1300\pi$$

$$\ell = \frac{65}{18}\pi \approx 11.34$$

Section 10.3

Grade: «grade»
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1. Draw circle A and using the Point on Circle command, place point C, D, and E on the circle. Construct segments BC and DE and measure their lengths. Drag the segments until their measurements are equal.

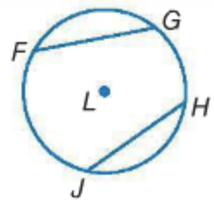
2. Measure the arc angles of their intercepted arcs. What do you notice?

3. What is your conjecture about the arc measures of congruent chords?

Theorem 10.2

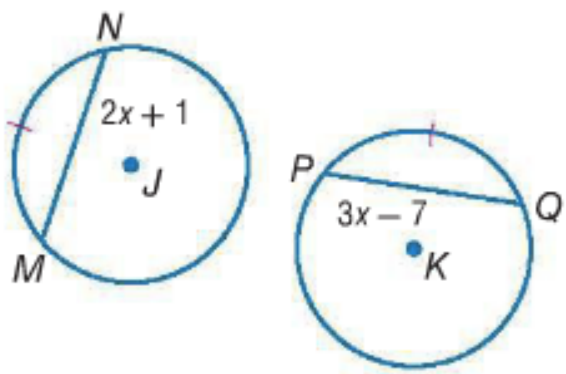
Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\overset{\frown}{FG} \cong \overset{\frown}{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.



1 Answer?

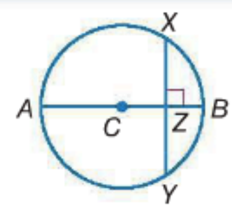
ALGEBRA In the figures, $\odot J \cong \odot K$ and $\widehat{MN} \cong \widehat{PQ}$. Find PQ.



Theorems

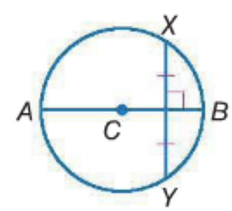
10.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

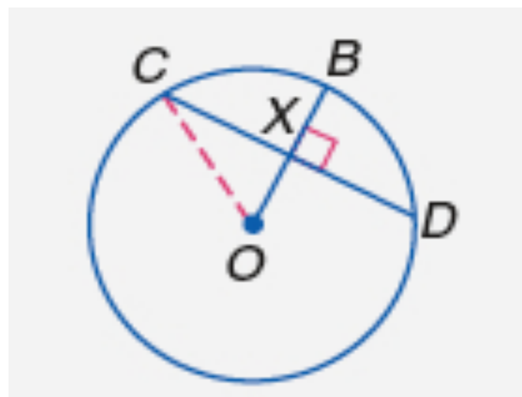
Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



Example 3 Radius Perpendicular to a Chord

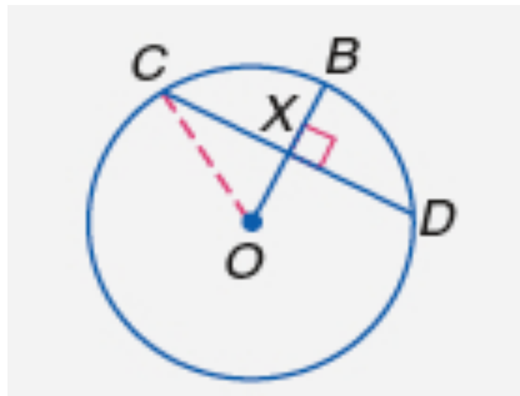
Circle O has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long.

a. If $m\overset{\frown}{CD} = 134$, find $m\overset{\frown}{CB}$.



2 Answer?

b. Find OX.



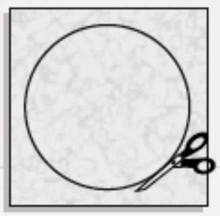
3 Answer?

Geometry Activity

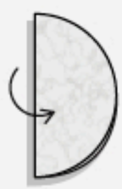
Congruent Chords and Distance

Model

Step 1 Use a compass to draw a large circle on patty paper. Cut out the circle.



Step 2 Fold the circle in half.



Step 3 Without opening the circle, fold the edge of the circle so it does not intersect the first fold.

Step 4 Unfold the circle and label as shown.

Step 5 Fold the circle, laying point V onto T to bisect the chord. Open the circle and fold again to bisect $\overline{WY}$. Label as shown.

Analyze

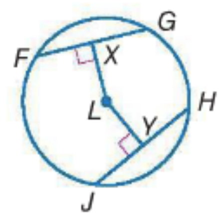
1. What is the relationship between $\overline{SU}$ and $\overline{VT}$? $\overline{SX}$ and $\overline{WY}$?
2. Use a centimeter ruler to measure $\overline{VT}$, $\overline{WY}$, $\overline{SU}$, and $\overline{SX}$. What do you find?
3. **Make a conjecture** about the distance that two chords are from the center when they are congruent.

Congruent Chords and Distance

Theorem 10.5

Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

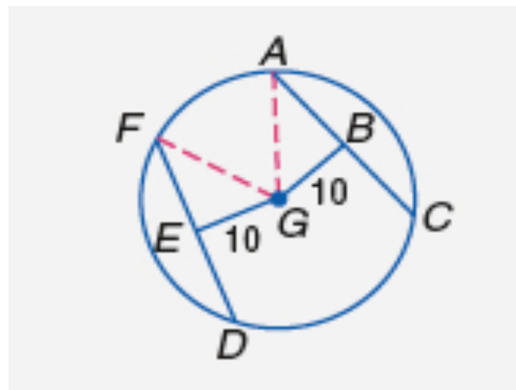
Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



You will prove Theorem 10.4 in Exercises 37 and 38.

Example 4 Chords Equidistant from Center

Chords $\overline{AC}$ and $\overline{DF}$ are equidistant from the center. If the radius of $\odot G$ is 26, find AC and DE.



4 Answer AC?

5 Answer DE?